THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 6

1. Let $(X, \mathfrak{T})$ be a topological space. Let $Z \subset Y \subset X$. Show that Z is compact in $(Y, \mathfrak{T}|_Y)$ if and only if Z is compact in $(X, \mathfrak{T})$.

2. Let X be a Hausdorff space and $C_1, C_2 \subset X$ be compact. Show that $C_1 \cap C_2$ is also compact.

3. * Show that every compact metric space X is separable.

   (Hint: For each $n \in \mathbb{N}$, consider the open covering $\{B(x, \frac{1}{n}) \mid x \in X\}$. Apply compactness to get a finite set of points. Then show that the union of the finite sets form a countable dense subset of X.)

   (Remark: Actually every compact metric space X is second countable.)